Short-term Finance, Long-term Growth: Evidence from a Loan Guarantee Program in Morocco

Preliminary

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Finance is important for firm growth, especially in emerging countries:

scarcity of long-term external finance and firm growth: i.e., Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), Arellano, Bai, and Zhang (2012), Moll (2014), Midrigan and Xu (2014), and more.

However, the role of short-term external finance on firm growth is less studied:

- Impact on firm growth?
- Mechanisms?
- Policies?

(1) We use a loan guarantee program (LGP) in Morocco to document the effect of short-term finance:

Fact 1 firms with guaranteed short-term loans decrease their cash ratio,

Fact 2 they expand their production scale persistently,

Fact 3 participation in the guarantee program is humped-shaped in firm size.

(2) We rationalize these findings in a heterogeneous-firm model:

- Short-term finance fulfills a firm's need for liquidity (i.e., working capital)and unproductive cash is reallocated to productive assets.
- ▶ It affects the firm's long-term scale in the presence of intertemporal distortions.
- Productive and medium-sized firms self-select into the program because of a flat entry cost.
- (3) We provide a counterfactual analysis:
 - increment in guaranteed ratio and lower entry cost

Aggregate gains: The aggregate gains from LGPs are elusive, both empirically and theoretically.

- We show that intertemporal distortions are key to generate a strong impact of the LGP both at the individual and aggregate level: self-financing cannot "undo" financial constraints
- In Morocco, intertemporal distortions are large
- $\rightarrow~$ The individual gains of entering a LGP are large.
- $\rightarrow~$ The aggregate gains of enlarging LGPs are substantial.
- Financial constraints interact with other frictions in the economy

Distributional effects:

increment in guaranteed ratio and lower entry cost both increase participation of small firms.

[Empirical Analysis]

Two databases:

- 1. Tamwilcom Guarantee Database: Background
 - Unit of observation: firm-year
 - 2012-2018
 - A database of 23,017 guarantees, mobilizing an amount of 22 billion dirham loans (about 2.2 billion Euros) extended to credit-constrained firms

2. Orbis Database:

- A commercial database by Bureau van Dijk (BvD)
- Firm-level balance sheet data collected from Office of Industrial and Commercial Property (OMPIC)
- About 1.58 million firms, representing the business world of Morocco

Merge the two databases: identify guaranteed firms in Orbis data Details

Endogeneity issue: Selection Bias

- Step 1: Multivariate Distance Matching
 - Find statistical twins for a guaranteed firm based on selected time-varying and observable variables that are relevant to selection into the program and outcomes. Details
- Step 2: Difference-in-Difference
 - Address firm-level unobservable fixed effects. Trend Inspection
- Goal: following the two steps, differences in outcome variables between treated and control firms can be effectively attributed to the guarantees.

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We apply the following regression to the matched sample.

$$\Delta Y_{igt} = \frac{\delta D_{it}}{\delta D_{it}} + \lambda_{gt} + \epsilon_{igt}$$

- *i* indexes firms, g indexes groups of one guaranteed firm with its matched controls
- ΔY_{igt} : change in outcome variable compared to the year prior to obtaining guarantee; $\Delta Y_{igt} = Y_{igs} - Y_{igt-1}$, where s = t + 1, t + 2, t + 3
- ▶ *D_{it}*: a dummy variable indicating if firm *i* is guaranteed in year *t*
- λ_{gt}: group and year fixed effects
- *ϵ_{igt}*: error term

Fact 1: Firm size increases persistently.

	(1)	(2)	(3)	(4)	(5)	(6)	-
-	Sa	les Growt	h	Ťotal	assets Gro	owth	-
	t+1	t+2	t+3	t+1	t+2	t+3	-
Guaranteed	0.140***	0.053	0.128^{*}	0.088***	0.079^{+}	0.149**	-
	(0.024)	(0.038)	(0.055)	(0.023)	(0.044)	(0.050)	5in Note:
N	18836	10770	5670	19150	11133	6015	=
adj. R ²	0.209	0.197	0.264	0.191	0.215	0.214	
Group × Year FE	Yes	Yes	Yes	Yes	Yes	Yes	
City × Year FE	Yes	Yes	Yes	Yes	Yes	Yes	_
standard errors ar	e clustere	ed at gro	up-year l	level. Sigi	nificance	level: +	p < 0.10,

Table: EFFECT OF TAMWILCOM GUARANTEE ON FIRM'S SALES AND TOTAL ASSETS

p < 0.05, ** p < 0.01, *** p < 0.001.

Other outcome variables: Costs of Employees Fixed Assets

Fact 2: Firms do not increase their cash holdings and increase their current liabilities.

	(1)	(2)	(3)	(4)	(5)	(6)	=
	Current	Liabilities	Growth	(Cash Grow	rth	-
	t+1	t+2	t+3	t+1	t+2	t+3	_
Guaranteed	0.138***	0.119**	0.194***	-0.061	-0.210^{+}	0.089	-
	(0.027)	(0.039)	(0.049)	(0.091)	(0.121)	(0.152)	5in Note:
N	19448	11262	6012	18766	10690	5816	_
adj. R ²	0.204	0.203	0.163	0.322	0.303	0.289	
Group × Year FE	Yes	Yes	Yes	Yes	Yes	Yes	
City × Year FE	Yes	Yes	Yes	Yes	Yes	Yes	_

Table: Effect of Tamwilcom Guarantee on Firm's Current Liabilities

Standard errors are clustered at group-year level. Significance level: + p < 0.10, * p < 0.05, ** p < 0.01, *** p < 0.001.

By size and age

Fact 3: Participation in the guarantee program is humped-shaped in firm size.

Figure: Participation Rate by Size



[Our Mechanism in a Simple Model]

A. Leontief Technology: $y_t = A[\min(k_t, a^{-1}l_t)]^{\alpha}$, subject to

- 1. Budget constraint: $k_t + c_t = n_t$
- 2. Working capital constraint: $w_t l_t \leq c_t + b_t$
- 3. Collateral constraint: $b_t \leq \theta k_t$

where

B. Entrepreneur's Static Optimization I: Trade-off:

$$\max_{c_{t},k_{t}} \underbrace{\psi(k_{t}) + (1-\delta)k_{t} + c_{t}}_{\pi_{t}} + \lambda_{t} (c_{t} + \theta k_{t} - ak_{t}) + \mu_{t} c_{t} + \gamma_{t} (n_{t} - k_{t} - c_{t})$$

$$MBK_{t} = \underbrace{1}_{\text{Unit Return of Saving}} + \underbrace{(\psi'(k_{t}) - \delta)}_{\text{Real Return of Production}} + \underbrace{\lambda_{t} (\theta - a)}_{\text{Shadow Return of Finance}}$$

$$MBC_{t} = \underbrace{1}_{\text{Unit Return of Saving}} + \underbrace{0}_{\text{Real Return of Production}} + \underbrace{\lambda_{t}}_{\text{Shadow Return of Finance}}$$

$$\psi_{t} = \psi(k_{t}) = (Ak_{t}^{\alpha-1} - ak_{t}) \text{ is production net of the wage bill.}$$

Simple Model: The Effects of Short-term Finance

C. The Effects on the Static Choices of Firms:

$$\frac{k_t}{n_t} = \frac{1}{1+a-\theta} \quad \& \quad \frac{c_t}{n_t} = \frac{a-\theta}{1+a-\theta}$$

Figure: Relationship between Optimal Choices and θ



A. Entrepreneur's Dynamic Optimization::

$$\nu(n_{t-1}) = \max_{k_t, c_t, d_t, n_t} \left\{ \frac{d_t^{1-\eta}}{1-\eta} + \beta \epsilon \nu(n_t) \right\}$$

subject to the constraints

$$(1-\tau)\pi_t-d_t-n_t\geq 0$$

Euler equation with $\beta(1 + r_{t+1}) = 1$:

$$\left(\frac{d_{t+1}}{d_t}\right)^{\eta} = \epsilon(1-\tau) \left(1 + \frac{\lambda_{t+1}}{\beta}\right)$$

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Simple Model: Growth Dynamics with Intertemporal Distortions

B. The Effects on the Dynamics of Firms' Long-run Growth with $\epsilon(1 - \tau) < 1$:

$$\lambda^{LT} = \frac{1}{\beta} \left(\frac{1}{\epsilon(1-\tau)} - 1 \right) \quad \& \quad \psi'(k^{LT}) - \psi'(k^{opt}) = (1+a-\theta)\lambda^{LT}$$

Figure: Dynamics of Firms' Long-run Growth



(a) Without Intertemporal distortons (b) With Intertemporal Distortions

Increase the supply of short-term finance (i.e., loan guarantee program):

A. Effects on the Static Choices of Firm Production and Financing:

- increase the firm's optimal choice of capital
- decrease the firm's optimal choice of cash holdings
- increase the firm's optimal output (sales)

B. Effects on the Dynamics of Firms' Long-run Growth with Intertemporal Distortions ($\epsilon(1 - \tau) < 1$):

increase the capital and production scale in the long-run

[Quantitative Model]

Heterogeneous production firms:

- Firm decides on capital, labor, debt, cash and dividends
- Firm faces exogenous exit risk & Concave utility
- Idiosyncratic productivity shocks

Short-term finance:

- Working capital constraint (financed with short-term debt or cash)
- Collateral constraint (borrow up to a proportion of capital value)
- Loan guarantee program (fixed cost of entry, multiplier on CC)

General equilibrium:

- Aggregate capital good producer
- Other households (supply labor and consume)



Model Fit

Moments	Data	Model
Output Dynamics		
1-year autocorrelation of output	0.89	0.89
3-year autocorrelation of output	0.69	0.71
5-year autocorrelation of output	0.53	0.56
Size ratio of entrant relative to average	17%	17%
Annual exit rate of firms	9.0%	9.0%
Financial Frictions		
Mean cash/asset ratio (non-guaranteed)	22%	22%
Mean cash/asset ratio (guaranteed)	9%	7%
Mean debt/asset ratio (non-guaranteed)	51%	38%
Mean debt/asset ratio (guaranteed)	64%	62%
Guaranteed loan/current liability ratio	22%	22%
Loan guarantee program		
Guaranteed loan commission fee	0.5%	0.5%
Percentage of loan guaranteed	60%	60%
Percentage of firms participating LGP	3.4%	3.8%

Table: Calibration of τ using the long-term effect of guarantee program

	(1)	(2)	(3)	(4)	(5)	(6)
	Sales	Growth (I	Data)	Sales	Growth (N	lodel)
	t+1	t+2	ť+3	t+1	t+2	t+3
Guaranteed	0.140^{***}	0.053	0.128^{*}	0.196***	0.177***	0.129***
	(0.024)	(0.038)	(0.055)	(0.005)	(0.008)	(0.009)
N	18836	10770	5641	480000	470000	460000
Adjusted R ²	0.209	0.197	0.263	0.551	0.488	0.461
Matched Group	Yes	Yes	Yes	No	No	No
Group × Year FE	Yes	Yes	Yes	No	No	No
Firm Controls	No	No	No	Yes	Yes	Yes
<u>Firm FE & Year FE</u>	No	No	No	Yes	Yes	Yes

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Figure: OPTIMAL DECISION RULES

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0.00 0.25 0.50 0.75 1.00 1.25

(b) Cash Policy

Net Worth

Distribution of firms

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Guaranteed Unguaranteed



Figure: PARTICIPATION RATE

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Counterfactual Analysis: Expansion of LGP

	Benchmark	C1	C2	C3	C4
Model Outcomes	$(\bar{\xi}_{bm}, \chi_{bm})$	$\bar{\xi}_{c1} = 0.173$	$\bar{\xi}_{c2} = 0.087$	$\chi_{c3} == 3.33$	$\chi_{c4} = 5.00$
Firm Financing					
LGP participation rate (%)	3.8	5.8	11.3	5.7	8.6
Guaranteed credit/total credit (%)	1.7	2.6	4.9	4.0	10.3
Mean cash/asset (guaranteed) (%)	6.5	6.5	6.5	3.1	0.09
Mean cash/asset (all firms) (%)	21.0	20.7	20.1	20.4	19.5
Mean debt/asset (guaranteed) (%)	62.2	62.2	61.9	70.1	83.9
Mean debt/asset (all firms) (%)	37.7	38.1	39.2	38.7	40.8
Distribution of Firm Financing					
Change in mean size (guaranteed, %)	-	-0.14	-2.14	-2.09	-4.78
Share of guaranteed credit (Q1, %)	1.24	1.26	1.29	1.72	3.15
Share of guaranteed credit (Q2, %)	3.79	3.79	3.89	5.37	8.10
Share of guaranteed credit (Q3, %)	20.65	20.38	20.81	24.42	27.77
Share of guaranteed credit (Q4, %)	74.60	74.57	74.00	68.49	60.97
Economic Outcomes					
Changes in Total Credit (%)	-	1.01	3.98	2.65	8.22
Changes in Aggregate TFP (%)	-	0.05	0.17	0.10	0.31
Changes in Total Output (%)	-	0.07	0.28	0.18	0.39
Changes in Total Consumption (%)	-	0.08	0.31	0.20	0.49
Changes in Total Welfare (%)	-	0.04	0.16	0.10	0.21

Table: The effect of a higher guarantee ratio (C4) under alternative assumptions on intertemporal distortions

	Benchmark	Lower τ	Higher ϵ
Model Outcomes	$(\tau = 0.02, \epsilon = 0.91)$	$(\tau = 0, \epsilon = 0.91)$	$(\tau = 0.02, \epsilon = 0.93)$
Economic Outcomes			
Changes in Aggregate TFP (%)	0.31	0.21	0.20
Changes in Total Output (%)	0.38	0.16	0.23
Changes in Total Welfare (%)	0.21	0.08	0.13

Note: This table reports the effects of policy C4 under different assumptions on τ and $\epsilon.$

- We study the effect of short-term finance on firm growth and its aggregate implications.
- Empirically, we find that firms with guaranteed loans expand their production scale and sharply decrease their cash holdings.
- Quantitatively, we show the gain of enlarging the loan guarantee programs (through higher guaranteed ratio or lower inspection costs) are substantial.
- Role of intertemporal distortions.

Thank You!

Appendix

Institutional Background Back to Data

- Tamwilcom, the credit guarantee agency in Morocco, facilitates bank credit to under-collateralized SMEs.
- We focus on two products related to working capital loans (Damane Exploitation and Damane Express) Details



We focus on two products related to working capital loans:

- Damane Exploitation: SMEs with sales below 175 million DH (about 17.5 million CHF)
- Damane Express (loan purpose: working capital): micro and small enterprises; loan size up to 1 million DH (100,000 CHF)



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Details on Damane Exploitation and Damane Express (2)



- Sales distribution of firms guaranteed under both products are smooth. We don't see lumps and gaps.
- Firms self-select into different programs based on the amount of their liquidity needs.

Back to Background

- Guaranteed firms in Tamwilcom database are paired with their balance sheet data in Orbis.
 - ▶ We have a rate of successful pairing of 49.3% (11,344 out of 23,017 CCGguaranteed firms are identified in Orbis database.)
 - Only 4000 guaranteed firms have data for the year where it is granted the guarantee.
 - The number drops further when we would like to have data of several consecutive years.
 - Only 4.3% of the CCG-guaranteed firms are in the final sample.
- Data attrition is consistent with existing literature in this field.
- Robustness check: inverse probability weights.

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Matching details:

- Nearest neighbor matching based on Mahalanobis distance of selected variables
- One guaranteed firm is matched with five non-guaranteed firms with replacement.
- Variables for matching: total assets (t-1, t-2, t-3), sales (t-1, t-2, t-3), current liabilities (t-1, t-2, t-3), cash (t-1, t-2, t-3) and firm age. (Note: year t refers to treatment year; all values are in log.)
- Exact match: year, firm size, firm sector, city
- Re-scale of weight: control firms are re-weighted based on their distance to the treated firm.
- Imposed caliper: we impose maximum distance allowed in matching to exclude control firms that are not sufficiently similar to treated firm.
- **Final sample**: 991 guaranteed firms are matched with 4,577 control firms. (9,218 firm-year observations are matched with 56,202 observations.)

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Sample Balancedness Check 1



- Good balancedness is observed.
- Standardized mean differences are close to zero.
- Variance ratios are close to one.

- Logit model: We evaluate the probability of obtaining a guarantee based on the variables used for the matching in both raw and matched samples.
- **Pseudo** R^2 : a drop indicates good balance in the sample.
- Purpose: the loss of explanatory power of selected variables towards treatment status after matching
- Indeed what we observe: It drops from 0.66 of the raw sample to 0.01 of the matched sample.

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Trend Inspection: Weighted Means



After the matching procedure, the two groups share parallel trends before the treatment year t.

The changes in the trends of weighted means give some preliminary evidence at the treatment effects.

	(1)	(2)	(3)	(4)	(5)	(6)
	Costs of	f Employees	Growth	Costs o	f Employe	es/Sales
	t+1	t+2	t+3	t+1	t+2	t+3
Guaranteed	0.116***	0.141***	0.151***	0.081	-0.016	0.052
	(0.016)	(0.024)	(0.042)	(0.076)	(0.011)	(0.041)
N	28618	22810	17076	28488	22685	16958
Adjusted R ²	0.301	0.319	0.333	0.447	0.368	0.370
Group × Year FE	Yes	Yes	Yes	Yes	Yes	Yes

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	(1)	(2)	(3)	(4)	(5)	(6)
	Fixe	d Assets Gro	owth	Fixed A	Assets/Tota	l Assets
	t+1	t+2	t+3	t+1	t+2	t+3
Guaranteed	0.139***	0.281***	0.283***	0.000	0.006	0.004
	(0.026)	(0.041)	(0.071)	(0.002)	(0.004)	(0.006)
N	27576	21862	16286	27576	21862	16286
Adjusted R ²	0.239	0.248	0.261	0.255	0.263	0.265
Group × Year FE	Yes	Yes	Yes	Yes	Yes	Yes

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Expansion in Current Liability

A guaranteed firm has an increase of 14.0% in current liabilities growth in the first year after the guarantee with persistent growth in the following two years.

	(1)	(2)	(3)	(4)	(5)	(6)
	Current	Liabilities	Growth	Current	Liabilities	Total Assets
	t+1	t+2	t+3	t+1	t+2	t+3
Guaranteed	0.140^{***}	0.144^{***}	0.224^{***}	-0.009	0.038	-0.014
	(0.016)	(0.025)	(0.052)	(0.009)	(0.044)	(0.023)
N	30382	24427	18487	30039	24069	18139
Adjusted R ²	0.395	0.411	0.413	-0.108	-0.052	0.068
Group × Year FE	Yes	Yes	Yes	Yes	Yes	Yes

 Table:

 Estimation Results of Tamwilcom Guarantee on Firm's Current Liabilities

Differential Effects Conditional on Size and Age

$$\Delta Y_{igt} = \delta D_{it} + \alpha D_{it} \times \log(X_{it}) + \lambda_{gt} + \epsilon_{igt}$$
(1)

Table: Effect of Tamwilcom Guarantee depending on Firm's Age and Size

	(1)	(2)	(3)	(4)	(5)	(6)
	S	ales Growt	h		Cash Grow	th
	t+1	t+1	t+1	t+1	t+1	t+1
Guaranteed	0.413***	0.955***	1.175***	-0.392*	-2.491***	-1.865***
	(0.084)	(0.280)	(0.310)	(0.190)	(0.570)	(0.519)
Guaranteed × <i>log</i> (Age)	-0.115***			0.116		
	(0.032)			(0.077)		
Guaranteed × $log(Total Assets)_{t-1}$		-0.052**			0.152***	
		(0.017)			(0.036)	
Guaranteed × $log(Sales)_{t-1}$			-0.066***			0.112***
			(0.019)			(0.033)
Ν	30136	30058	30136	30083	30083	29773
Adjusted R ²	0.416	0.417	0.417	0.385	0.385	0.384
Group × Year FE	Yes	Yes	Yes	Yes	Yes	Yes

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Production:

$$y_{i,t} = z_{i,t} k_{i,t}^{\alpha} l_{i,t}^{\nu} e^{1-\alpha-\nu}, \quad \alpha+\nu < 1$$

$$\log(z_{i,t}) = \rho_z \log(z_{it-1}) + \sigma_z \varepsilon_{i,t}$$

Short-term financing:

Working capital constraint:

$$w_t l_{i,t} \leq c_{i,t} + b_{i,t}$$

Collateral constraint (?):

$$b_{i,t} \leq \bar{b_{i,t}} \equiv \theta_0 k_{i,t} + \theta_1 \Psi(k_{i,t}) = \left[\theta_0 + \theta_1 \frac{\Psi(k_{i,t})}{k_{i,t}}\right] k_{i,t}$$

Loan guarantee program for SMEs:

$$b_{i,t} \le \begin{cases} (1 + (\chi - 1)s) \left(\theta_0 k_{i,t} + \theta_1 \Psi(k_{i,t})\right) & \text{if } F = A\\ \theta_0 k_{i,t} + \theta_1 \Psi(k_{i,t}) & \text{if } F = N \end{cases}$$

First period (maximize total profit):

$$\pi^*(z_{i,t}, n_{i,t-1}, F_{i,t}) = \max_{k,c,l} \left\{ z_{i,t} k_{i,t}^{\alpha} l_{i,t}^{\nu} - w_t l_{i,t} + (1-\delta) q_t k_{i,t} + (1+r_t) c_{i,t} - r_t b_{i,t} - F_{i,t} \cdot \mu \tilde{b}_{i,t} \right\}$$

subject to:

$$n_{i,t-1} = q_t k_{i,t} + c_{i,t}$$
$$w_t l_{i,t} \le c_{i,t} + F_{i,t} \cdot (1 + (\chi - 1)s) \left(\theta_0 k_{i,t} + \theta_1 \Psi(k_{i,t}) \right) + (1 - F_{i,t}) \cdot \left(\theta_0 k_{i,t} + \theta_1 \Psi(k_{i,t}) \right)$$
$$\tilde{b}_{i,t} = (\chi - 1)s(w_t l_{i,t} - c_{i,t})$$
$$b_{i,t} = w_t l_{i,t} - c_{i,t}$$

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Value function:

$$\nu(z_{i,t}, n_{-1}, F_{i,t}) = \max_{d_{i,t}} \left\{ \frac{d_{i,t}(z_{i,t}, n_{i,t-1}, F_{i,t})^{1-\eta}}{1-\eta} + \epsilon \beta E_z[\nu(z_{i,t+1}, n_{i,t})] \right\}$$

Net worth accumulation:

$$n_{i,t}(z_{i,t}, n_{i,t-1}, F_{i,t}) = \pi^*(z_{i,t}, n_{i,t-1}, F_{i,t}) - d_{i,t}(z_{i,t}, n_{i,t-1}, F_{i,t}) - \xi_{i,t}$$

LGP fixed cost threshold:

$$\xi^{*}(z_{i,t}, n_{i,t-1}) = \frac{\pi^{*}(z_{i,t}, n_{i,t-1}, A) - \pi^{*}(z_{i,t}, n_{i,t-1}, N)}{w_{t}}$$

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Other Household:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \theta \frac{L_t^{1+\omega}}{1+\omega} \right)$$

subject to:

$$C_t + \frac{1}{1 + r_t} B_t \le B_{t-1} + W_t L_t$$

Capital Good Producer:

Technology:

$$\Phi(I_t/K_t)K_t = \left(\frac{\delta/\phi}{1-1/\phi} \left(\frac{I_t}{K_t}\right)^{1-1/\phi} - \frac{\delta}{\phi-1}\right)K_t$$

Price of capital:

$$q_t = \frac{1}{\Phi'(I_t/K_t)} = \frac{I_t/K_t}{\delta}^{1/\phi}$$

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Parameter	Description	Value
Firms		
α	Capital coefficient	0.21
ν	Labor coefficient	0.64
δ	Capital depreciation	0.10
ϕ	Capital adjustment cost	4.0
Households		
β	Discount factor	0.96
η	Elasticity of intertemporal substitution	1
$\dot{\theta}$	Leisure preference	2
ω	Inverse Frisch	0.5

Parameter	Description	Value
Output Dynamics		
ρ_z	Persistence of TFP shock	0.90
σ_z	Volatility of TFP shock	0.06
<u>n</u> 0	Net worth of entrants	0.07
e	Survival rate	0.91
τ	Net worth erosion	0.02
Financial Frictions		
S	Share of formal bank loans	0.20
θ_0	Collateral constraint (size-irrelevant)	0.01
θ_1	Collateral constraint (size-dependent)	0.26
Y	Collateral constraint (size-dependent)	1.35
Loan guarantee prograi	m	
μ	Guaranteed loan commission fee	0.5%
X	Multiplier of LGP on loans	2.5
Ë	Upper bound of LGP fixed cost	0.26

How does the guarantee help firm growth? Selection vs Growth



Figure: Distribution of Firm's Long-term Scale

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